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Chapter 0

About this book

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Disclaimer.
This was a work in progress. The author has ceased active involvement with Mercury and there are (currently) no plans to continue work on it.

What is this book for?
A basis for courses teaching or using Mercury. A tutorial for experienced programmers wishing to learn Mercury.

What version of Mercury is used in the book?
Any recent release-of-the-day (ROTD) compiler will do. The text as of today (10 April 2005) will work fine for Mercury release 0.12.

Who is it aimed at?
Students who have completed at least one year of a computer science degree, been exposed to a strongly, statically typed functional programming language (e.g., ML, Haskell, Miranda), and have some reasonable programming experience.

What is this book about?
This book is a tutorial on programming in Mercury. The book should provide the reader with enough information to become a competent Mercury programmer and knowledge sufficient to understand the Mercury Reference Manual, the Mercury Library Reference Manual and the Mercury Users’ Guide. The book will describe key features of Mercury in detail, including the type system, the mode and determinism systems, and programming with
nondeterminism.

**What is this book not about?**
This book will not teach you how to program in general, nor will it teach you a great deal of theory. It will not teach you how type inference works; it will (probably) not teach you about Herbrand universes; it will not include an extended treatise on IO in declarative programming languages; it will not (in the first place) cover black-belt Mercury programming techniques (e.g. dynamic type casts for non-ground types).

**Where is the introduction?**
I plan to write the introduction last. It will include a brief description of “what Mercury is all about” and an explanation of what the book is trying to do.
Chapter 1

Mercury By Example

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This chapter aims to convey through examples a basic feeling for how Mercury works. Because this is the first chapter, we may gloss over some fine detail and allow ourselves a certain latitude in precision, but this will not worry us as we will correct these deficiencies in later chapters. The approach taken here is to start by presenting the “obvious” solution to a problem and then introduce features of Mercury that allow for more elegant or efficient programs.

1.1 Hello, World!

It is slightly unfortunate that the “Hello, World!” program introduces no less than three advanced Mercury concepts, but since tradition dictates that tutorial texts start with “Hello, World!” we’ll just have to jump straight in with the knowledge that things will get easier thereafter.

We’ll start by presenting the complete program which we’ll assume we’ve typed into a file called hello.m:

```mercury
:- module hello.
:- interface.
:- import_module io.
:- pred main(io::di, io::uo) is det.
:- implementation.
```
main(IOState_in, IOState_out) :-
    io.write_string("Hello, World!
", IOState_in, IOState_out).

Let’s go through this line by line.

:- module hello.

Every module must start with a declaration like this giving the name of the module; the Mercury compiler will expect a module called hello in a file called hello.m.

:- interface.

An interface declaration introduces the part of module describing the things we are going to export (i.e., allow users of the module to see).

:- import_module io.

This import_module declaration says that we need to use some of the names exported by the io module, which is part of the standard Mercury library.

:- pred main(io::di, io::uo) is det.

This says that we are going to define a predicate called main with two arguments of type io (which happens to be defined in the io module), the first of which is a destructive input argument, and the second of which is a unique output argument; the is det part tells us that main is a deterministic predicate — that is, main always succeeds and will always compute the same output given the same input (we will see later that some Mercury predicates can fail and some can have more than one solution).

Every Mercury program needs to export a predicate called main, which the compiler takes as the starting point for the Mercury program as a whole.

:- implementation.

Everything after an implementation declaration is considered private implementation detail not visible to other users of the module.
1.1. **HELLO, WORLD!**

```prolog
main(IOState_in, IOState_out) :-
  io.write_string("Hello, World!\n", IOState_in, IOState_out).
```

Finally, we have a clause defining `main`. A clause comprises a head and body separated by a `:-` symbol. The head of this clause tells us that this is a definition for `main` and names its two arguments `IOState_in` and `IOState_out`. The body of the clause, which is executed when `main` is called, consists of a single goal calling `io.write_string` (i.e., the `write_string` predicate defined in the `io` module) with a message (Mercury interprets the `\n` sequence in the string argument as a literal newline character) and the two `IOState` arguments.

Program variables in Mercury always start with a capital letter or underscore, while names of predicates, types, modules and so forth do not.

We can compile and run `hello.m` as follows ($ indicates the command line prompt):

```
$ mmc --make hello
Making Mercury/int3s/hello.int3
Making Mercury/cs/hello.c
Making Mercury/os/hello.o
Making hello
$ ./hello
Hello, World!
```

Et voilà! (By default, `mmc --make` will construct a local Mercury directory, if necessary, to hold intermediate files generated during compilation.)

At this point the reader is probably wondering about the meaning of the `io` type arguments `IOState_in` and `IOState_out`, and the strange `di` and `uo` argument modes. The short answer is that every predicate that performs I/O has to have an `io` type input argument describing the state of the world at the time the predicate is called and an `io` type output argument describing the state of the world after the call. This is how Mercury allows programs to communicate with the outside world without hidden side effects which would compromise its mathematical integrity.

Because it doesn’t make much sense to try reusing an old `io` state — you can’t go back in time — `io` states are unique. This is where the `di` and `uo` argument modes come in: they are just like the ordinary `in` and `out` argument modes we’ll meet in the next section, except that they also specify
uniqueness. The Mercury compiler will not allow programs to copy unique values or reuse dead ones: there is only ever one “live” I/O state at a time in a running Mercury program. This property guarantees that I/O operations occur in the intended order.

So what if we want to do more than one I/O operation? In this case we have to give names to each of the intermediate io states:

```mercury
main(IOState_in, IOState_out) :-
    io.write_string("Hello, ", IOState_in, IOState_1),
    io.write_string("World!", IOState_1, IOState_2),
    io.nl(IOState_2, IOState_out).
```

The first call to `io.write_string` takes `IOState_in` as an input, destroys it in the process of writing its string argument, and produces `IOState_1` as its result. Then the second call to `io.write_string` destroys `IOState_1` and produces `IOState_2`. Finally, `io.nl` (which just writes out a newline), destroys `IOState_2` and returns `IOState_out`, which is the result of the call to `main`.

Naming all these intermediate states quickly becomes tedious, so Mercury provides us with syntactic sugar in the form of `state` variables:

```mercury
main(!IO) :-
    io.write_string("Hello, ", !IO),
    io.write_string("World!", !IO),
    io.nl(!IO).
```

This code is transformed by the compiler into something equivalent to the preceding example: each occurrence of the `!IO` state variable actually stands for two normal variables, which are given intermediate names in the “obvious” way (the full details of the state variable transformation can be found in the Mercury Reference Manual). Note that there is no special significance in the name `IO`, which we merely use by convention.

Points to remember

- A module starts with a `module` declaration and is followed by an `interface` section and an `implementation` section.
- The interface section declares the things that are exported by the module.
1.1. HELLO, WORLD!

- All declarations start with a `:-` symbol.
- Declarations and clauses always end with a full stop.
- We have to import a module before we can use things exported by that module.
- Every Mercury program must export a predicate called `main`.
- Predicates are defined by clauses, which go in the implementation section.
- Variable names start with a capital letter; names of modules, types, predicates and so forth start with a lower-case letter or use symbols (e.g., the `int` module defines `+` to mean integer addition).
- Every predicate that performs I/O must have an `io::di` argument and an `io::uo` argument and must be deterministic.
- We use state variables to avoid having to explicitly name every `io` state.
1.2 The Fibonacci numbers

Another great computer science tradition is defining a function to calculate numbers in the Fibonacci series: 1 1 2 3 5 8 13 21 34...where, apart from the leading 1s, each number is the sum of the preceding two.

As before, we’ll start by showing a complete program and then look at the interesting parts in more detail.

:- module fib.
:- interface.
:- import_module io.

:- pred main(io::di, io::uo) is det.

:- implementation.
:- import_module int.

:- pred fib(int::in, int::out) is det.

fib(N, X) :-
  ( if N =< 2 then X = 1
  ~ else fib(N - 1, A), fib(N - 2, B), X = A + B
  )

main(!IO) :-
  fib(17, X),
  io.write_string("fib(17, ", !IO),
  io.write_int(X, !IO),
  io.write_string("
")
 ).

Building and running fib.m, we find that...

$ mmc --make fib
Making Mercury/int3s/fib.int3
Making Mercury/cs/fib.c
Making Mercury/os/fib.o
Making fib
$ ./fib
fib(17, 1597)
1.2. THE FIBONACCI NUMBERS

The first thing to note is the import_module declaration at the start of the implementation section. We need to import the int module from the Mercury standard library because it defines all the operations on integers, such as addition and comparison. We import it in the implementation section rather than the interface section because that’s the only place we refer to names defined in the int module.

Next the declaration

:- pred fib(int::in, int::out) is det.

says that we are going to define a predicate fib taking two int arguments, an input and an output, which always succeeds and always computes the same output given the same input.

fib(N, X) :-
    ( if  N =< 2
        then X = 1
        else fib(N - 1, A), fib(N - 2, B), X = A + B
     ).

The body of the fib definition uses an if-then-else goal to decide what to do; the else part is not optional; and the whole thing appears in parentheses. The condition N =< 2 succeeds if N is less than or equal to 2 and fails otherwise (we’ll learn more about semidet predicates like =< in later examples). If N =< 2 then the unification X = 1 is executed. Otherwise fib is called twice to compute the preceding two Fibonacci numbers in A and B, and X is unified with their sum (note that we don’t need to declare A and B anywhere).

Finally we have

main(!IO) :-
    fib(17, X),
    io.write_string("fib(17, ", !IO),
    io.write_int(X, !IO),
    io.write_string("\n", !IO).

which calls fib(17, X), unifying X with the result of computing the 17th Fibonacci number, then writes out the answer.
Now, just as \( N - 1 \) computes \( N \) minus 1 and \( A + B \) computes the sum of \( A \) and \( B \), it is possible to define \( \text{fib} \) so that \( \text{fib}(N) \) computes the \( N \)th Fibonacci number:

\[
:- \text{func} \; \text{fib}(\text{int}) = \text{int}.
\]

\[
\text{fib}(N) = X :-
( \text{if} \; N \leq 2 \text{ then } X = 1 \text{ else } X = \text{fib}(N - 1) + \text{fib}(N - 2) )
\]

The \text{func} declaration introduces \( \text{fib} \) as a \text{function} with an \text{int} argument computing an \text{int} result. Mercury assumes that the input arguments to a function have mode \text{in}, the result has mode \text{out}, and that the function as a whole is \text{det}.

Functions are also defined using clauses, the only difference being that the head of a function clause takes the form \text{FuncCall} = \text{Result}.

Having redefined \( \text{fib} \) as a function, we also have to change how it is called in \text{main}:

\[
\text{main}(!\text{IO}) :-
( \text{io.write_string("fib(17) = ", !\text{IO}),}
\text{io.write_int(fib(17), !\text{IO}),}
\text{io.nl(!\text{IO}).}
\)

Here is one last refinement we might like to make to our definition of \( \text{fib} \):

\[
\text{fib}(N) = \text{(if } N \leq 2 \text{ then 1 else } \text{fib}(N - 1) + \text{fib}(N - 2) \text{ ).}
\]

By using an \text{if-then-else expression} we can move the entire body into the head of the clause. Since the clause now has an empty body, Mercury requires that we omit the \):- part.

It is worth noting that all three of our definitions are computationally identical and there is no difference in the code generated by the Mercury compiler. Whether to use a predicate or function is largely a matter of taste; a good rule of thumb is that if a predicate has a single output argument, and is deterministic, then it is probably better expressed as a function.
Points to remember

- Import modules in the implementation section only if they are only referred to in the implementation section.

- Non-unique arguments (typically non-io arguments) normally use the in and out argument modes.

- You can use functions to make your code more readable.

- if-then-else can be used as a goal and as an expression; it should appear in parentheses and the else part is mandatory.

- If a clause of a predicate or a function has an empty body (because all the computation is described in the head), then the :- must be omitted.
1.3 Simple input

We can extend our Fibonacci program to read in $N$ rather than have it hard-coded by changing the definition of `main`:

```mercury
:- import_module list, string.

main(!IO) :-
    io.read_line_as_string(Result, !IO),
    ( if
        Result = ok(String),
        string.to_int(string.strip(String), N)
    then
        io.format("fib(%d) = %d\n", [i(N), i(fib(N))], !IO)
    else
        io.format("That isn’t a number...\n", [], !IO)
).
```

The `list` and `string` standard library modules are imported in the implementation section because we use them in the definition of `main` (the `io` module has already been imported in the interface section).

`main` begins by calling `io.read_line_as_string`, which reads a whole line of input up to and including the next newline character. If all goes well then `Result` ends up unified to a value `ok(String)`, where `String` is the string of characters read in and `ok` is a “tag” known as a data constructor. Other possibilities for `Result` are `eof`, indicating the end-of-file has been reached, and `error(ErrorCode)`, indicating that something went wrong.

Then an `if-then-else` decides what to do. The `if` condition succeeds if `Result` is an `ok` value (unifying `String` with the argument) and if `string.to_int(string.strip(String), N)` succeeds. The `string.strip` function returns its argument minus any leading and trailing whitespace, including the terminating newline character, while the predicate `string.to_int` succeeds if its first argument is a string of decimal digits (unifying `N` with the corresponding number), and fails otherwise.

It’s worth taking a slightly closer look at the unification in the `if` part of the `if-then-else` goal:

```
Result = ok(String)
```
We know that Result has a value at this point, so this kind of unification is known as a *deconstruction*: it only succeeds if the value in Result matches the pattern `ok(something)`, in which case it unifies String with the *something*.

The then and else arms of the if-then-else goal call the `io.format` predicate, which is rather like C’s printf function. The first argument is a format string (where %d indicates a decimal integer, %f indicates a floating point value, %s indicates a string, and %c indicates a character) and the second argument is a list of the corresponding values in [brackets], tagged with i, f, s or c for int, float, string or char values respectively. Thus in

\[
\text{io.format("fib(\%d) = \%d\n", [i(N), i(fib(N))], !IO)}
\]

the int N is printed instead of the first %d in the format string and the int result of fib(N) is printed instead of the second %d. If N = 17 we’d expect this goal to output

\[
fib(17) = 1597
\]

Just as the Mercury compiler doesn’t know how to do anything with integers unless you import the int module, it doesn’t know anything about lists unless you import the list module. So if you want to use `io.format` then you also have to import the list and string modules.

Okay, so far so good. What if we want to input more than one number in a loop? That’s easy to arrange:

\[
\text{main(!IO) :-}
\]

\[
\text{io.read_line_as_string(Result, !IO),}
\]

\[
( \text{ if } \]
\[
\text{Result = ok(String),}
\]

\[
\text{string.to_int(string.strip(String), N)}
\]

\[
\text{then}
\]

\[
\text{io.format("fib(\%d) = \%d\n", [i(N), i(fib(N))], !IO),}
\]

\[
\text{main(!IO)}
\]

\[
\text{else}
\]

\[
\text{io.format("I didn’t expect that...\n", [], !IO)}
\]

\[
) .
\]
CHAPTER 1. MERCURY BY EXAMPLE

Now the then branch calls main recursively to read in another number. Being a declarative language, recursion is Mercury’s only looping construct. However, as with any decent declarative language compiler worth its salt, tail recursion like this is just as efficient as a while or for loop in any other language.

Firing up the compiler we get

```
$ mmc --make fib
Making Mercury/int3s/fib.int3
Making Mercury/cs/fib.c
Making Mercury/os/fib.o
Making fib
$ ./fib
  |
  10
fib(10) = 55
  |
  17
fib(17) = 1597
  |
  20
fib(20) = 6765
  |
^D
```

I didn’t expect that...

(The |s indicate input from the user and don’t actually appear on the screen. ^D indicates the user typing Ctrl-D to close the input stream; Windows users should use Ctrl-Z.)

It would be good to handle the end-of-file condition more gracefully. The obvious way to do that is to add another case to the if-then-else:

```
main(!IO) :-
io.read_line_as_string(Result, !IO),
  ( if
    Result = eof
    then
      io.format("bye bye...\n", [], !IO)
    else if
      Result = ok(String),
      string.to_int(string.strip(String), N)
    then
      io.format("fib(%d) = %d\n", [i(N), i(fib(N))], !IO),
      main(!IO)
  ).
```
Observe the deconstruction test of Result with \texttt{eof} — the \texttt{eof} data constructor has no argument list and indeed it is a syntax error to write \texttt{eof()}.

Before we leave our \texttt{fib} example, let us introduce Mercury’s \textit{switch} goals. A switch goal is rather like C’s \textit{switch} statement and consists of a set of alternatives testing a given variable against different possible values it might have. Here is \texttt{main} rewritten to use a switch goal:

```mercury
main(!IO) :-
    io.read_line_as_string(Result, !IO),
    ( Result = eof,
      io.format("bye bye...\n", [], !IO)
    ;
    Result = ok(String),
      ( if string.to_int(string.strip(String), N)
          then io.format("fib(\%d) = \%d\n", [i(N), i(fib(N))], !IO)
          else io.format("that isn’t a number\n", [], !IO)
      ),
      main(!IO)
    ;
    Result = error(ErrorCode),
      io.format("\%s\n", [s(io.error_message(ErrorCode))], !IO)
    ).
```

A sequence of goals separated by semicolons is called a \textit{disjunction} (the semicolon is usually pronounced “or”). If each \textit{disjunct} deconstructs a particular variable against a set of mutually exclusive possibilities, then the disjunction as a whole is a switch. In general it is good style to use a switch rather than a sequence of \texttt{if-then-else} goals since then, in most cases, the Mercury compiler will tell you if you’ve forgotten a possibility or counted the same possibility twice.

**Points to remember**

- Some types (such as the result type of \texttt{io.read_line_as_string}) use different data constructors for different values. These values can be tested using deconstruction unifications.
• A data constructor with no arguments, such as `eof`, is not followed by an argument list.

• `io.format` is Mercury’s version of C’s `printf`. To use it you must import `list` and `string` as well as the `io` module.

• You should omit the parentheses around an `if-then-else` that immediately follows the `else` part of another `if-then-else`.

• A disjunction is a sequence of goals separated by semicolons.

• A switch is a disjunction where each disjunct tests a particular variable against a different possibility. Where applicable, switches are generally preferable to `if-then-elses`. 
1.4 rot13

Let’s move on to a different example. This time we are going to implement the rot13 “encryption” algorithm, which works by rotating the Roman alphabet by 13 places — in other words, \texttt{abcdefghijklmnopqrstuvwxyz} in the input becomes \texttt{nopqrstuvwxyzabcdefghijklm} in the output. Decryption is simple: just use rot13 a second time! While rot13 has the cryptographic strength of damp tissue paper, it is sometimes useful for obscuring information in an e-mail that the recipient may not yet wish to know, such as who won the Grand Final.

Here’s a first cut at a solution:

```prolog
:- module rot13.
:- interface.
:- import_module io.

:- pred main(io::di, io::uo) is det.

:- implement_module char, list, string.

main(!IO) :-
  io.read_char(Result, !IO),
  (Result = ok(Char),
   io.write_char(rot13(Char), !IO),
   main(!IO)
  ; Result = eof
  ; Result = error(ErrorCode),
   io.format("%s
", [s(io.error_message(ErrorCode))], !IO)
  ).

:- func rot13(char) = char.

rot13(Char) = (if Char = 'a' then 'n'
   else if Char = 'b' then 'o'
   ... else if Char = 'z' then 'm'
   else if Char = 'A' then 'N'
   ...).
```
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else if Char = 'B' then 'O'
...
else if Char = 'Z' then 'M'
else Char
).

While this plainly works:

$ mmc --make rot13
Making Mercury/int3s/rot13.int3
Making Mercury/cs/rot13.c
Making Mercury/os/rot13.o
Making rot13
$ ./rot13
| Port Adelaide beat the Brisbane Lions 113 to 73 in the Grand Final.
| Cbeg Nqrynqvr orng gur Gfvfonar Yvbafr 113 gb 73 va gur Tenaq Svany.
| Cbeg Nqrynqvr orng gur Gfvfonar Yvbafr 113 gb 73 va gur Tenaq Svany.
Port Adelaide beat the Brisbane Lions 113 to 73 in the Grand Final.

it's hardly going to win prizes for elegance or efficiency. A more experienced Mercury programmer might code rot13 like this:

:- func rot13(char) = char.
rot13(CharIn) = ( if rot13_2(CharIn, CharOut) then CharOut else CharIn ).

:- pred rot13_2(char::in, char::out) is semidet.
rot13_2('a', 'n').
rot13_2('b', 'o').
...
rot13_2('z', 'm').
rot13_2('A', 'N').
rot13_2('B', 'O').
...
rot13_2('Z', 'M').

There are three new things here: the semidet determinism category; clauses with the arguments already “filled in”; and using more than one clause to define a predicate.
First off, the semidet determinism category means that rot13_2 will, for any given input, either fail or have a single solution for the output. Looking at the code we might guess (correctly) that rot13_2('z', X) should succeed unifying X = 'm' (and never anything else), while rot13_2('7', X) would fail.

Secondly, a clause like

\[
\text{rot13_2('a', 'n').}
\]

is just syntactic sugar for

\[
\text{rot13_2(V1, V2) :- V1 = 'a', V2 = 'n'.}
\]

Since we know from the pred declaration for rot13_2 that V1 is an input and V2 an output, the unification V1 = 'a' must be a deconstruction test and, if that succeeds, then the construction unification V2 = 'n' is carried out (a construction unification always succeeds because the "destination" variable, V2 in this case, does not have a value before this point).

Finally, a sequence of clauses is syntactic sugar for a single clause whose body is a disjunction. Hence our code is transformed by the compiler into this:

\[
\text{rot13_2(V1, V2) :- ( V1 = 'a', V2 = 'n' ; V1 = 'b', V2 = 'o' ; \ldots ; V1 = 'z', V2 = 'm' ; V1 = 'A', V2 = 'N' ; V1 = 'B', V2 = 'O' ; \ldots ; V1 = 'Z', V2 = 'M' ).}
\]

The astute reader will immediately identify this as a switch on V1 because each disjunct tests the input V1 for a different possible value. One of the good things about switches is that the Mercury compiler will generate very efficient code for them, using a lookup-table or hash-table perhaps, which will certainly out-perform the long if-then-else chain in our first attempt.

(As an aside, the is semidet determinism declaration for rot13_2 tells the Mercury compiler that this predicate is expected to fail in some cases, so
it will not warn us about missing possible values for the first argument. In the `fib` program the switch had to be exhaustive, so it could not fail, because `main` was declared to be `det`. Had we missed a possible case out of that switch, the compiler would have reported the missing case as an error. Similarly, because `rot13_2` cannot have more than one solution for any input, the compiler will report an error if we have duplicate clauses matching the same input. The compiler can’t warn us about such problems if we use `if-then-else` chains. Try it and see what happens."

Note that if we felt so inclined, we could make `rot13_2` deterministic by including the translation of every possible character! Of course, there are many ways of coding `rot13` and while our implementation may not be the most concise, it is quite efficient and very easy to understand. Either way, our aim here was to look more closely at the concept of semideterminism and introduce the technique of making code more readable by using multiple clauses to define a predicate or function.

**Points to remember**

- Literal character values in Mercury are normally enclosed in single quotes. Some characters which the Mercury parser would normally expect to see used as infix function symbols, such as `+` and `*`, also need to be enclosed in parentheses: `( '+' )` and `( '*' )`. Full details of how special characters should be written can be found in the Mercury Reference Manual.

- The `semidet` determinism category means that a predicate can have at most one solution for a given set of inputs (it fails if it has no solution for the given inputs).

- Semideterministic predicates therefore often appear in the conditions of `if-then-else` goals.

- “Filling in” the arguments of a clause is just shorthand for omitting the equivalent unification goals.

- The compiler views a predicate or function definition comprising several clauses as a single disjunction. Multiple clauses are often easier to read than a large disjunction.
1.5 Cryptarithms

In this example we introduce nondeterminism to demonstrate a little of what differentiates Mercury from conventional programming languages. A cryptarithm is an equation (usually just an addition) where the digits of each number have been replaced by letters, $\text{DOG} + \text{ANT} = \text{CAT}$ for example; a solution is a mapping from letters to digits that satisfies the equation. Leading letters cannot stand for zero and each letter must stand for a distinct digit. Here's a Mercury program to solve this particular cryptarithm:

```mercury
:- module crypt.
:- interface.
:- import_module io.

:- pred main(io::di, io::uo) is cc_multi.

:- import_module int, list, string.

main(!IO) :-
  io.format("DOG + ANT = CAT\n", [], !IO),
  ( if
    Ds0 = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9], C0 = 0,
    pick(Ds0, G, Ds1),
    pick(Ds1, T, Ds2), S1 = G+T+C0, T = S1 mod 10, C1 = S1/10,
    pick(Ds2, 0, Ds3),
    pick(Ds3, N, Ds4), S2 = 0+N+C1, A = S2 mod 10, C2 = S2/10, A \= 0,
    pick(Ds4, D, Ds5),
    pick(Ds5, A, Ds6), S3 = D+A+C2, C = S3 mod 10, C3 = S3/10, C \= 0,
    pick(Ds6, C, _),
    C3 = 0
  then
    DOG = 100 * D + 10 * O + G,
    ANT = 100 * A + 10 * N + T,
    CAT = 100 * C + 10 * A + T,
    io.format("%d + %d = %d\n", [i(DOG), i(ANT), i(CAT)], !IO)
  else
    io.format("has no solutions\n", [], !IO)
  ).

:- pred pick(list(int)::in, int::out, list(int)::out) is nondet.
```
CHAPTER 1. MERCURY BY EXAMPLE

pick([X | Xs], X, Xs).
pick([X | Xs], Y, [X | Zs]) :- pick(Xs, Y, Zs).

At this early stage we won’t explain exactly how this program works. Instead, we will describe what it does and leave the technical explanation to later chapters.

The interesting part of this program is the condition of the if-then-else goal. The trick to understanding this code fragment is to consider it as a set of constraints on a solution rather than as a series of computations. (Concentrating on the “what” rather than the “how” is a hallmark of declarative programming.)

Ds0 is the list of digits from 0 to 9, Ds1 is Ds0 with the digit for G removed, Ds2 is Ds1 with the digit for T removed, Ds3 is Ds2 with the digit for O removed, and so forth. In this way we ensure that each of D, O, G, A, N, T and C are different.

The digits for each letter are selected nondeterministically using the pick predicate. The goal pick(Ds0, G, Ds1), for instance, picks a digit for G from Ds0 and leaves the remaining digits in Ds1. Since Ds0 contains ten members, there are ten possible solutions for G, and Ds1 will have nine members.

C1 is the carry from the units column, C2 is the carry from the tens column, C3 is the carry from the hundreds column; we introduce C0 as a “carry in” of zero to give the program a regular structure.

The goal S1 = G + T + C0, T = S1 mod 10 provides the constraint for the units column. We do the same thing for the tens and hundreds columns.

The goals A \= 0 and D \= 0 ensure that we don’t have any zeroes in the hundreds column (\= means “not equal to”), while C3 = 0 ensures that there is no carry left over from the hundreds column.

Eventually either a set of solutions to the pick goals will be found that is consistent with the other constraints or, if no such set exists, the condition of the if-then-else will fail.

At this point we can explain the cc_multi determinism category for main, which is different to det which we’ve used in all the preceding examples. cc_multi stands for committed choice multideterministic, which means that although this predicate may have multiple possible answers (there may be
multiple solutions to \texttt{main}'s \texttt{if-then-else} condition) we will only be getting one of them! This extra constraint is necessary to allow a predicate with more than one possible solution to perform I/O; since we’re not allowed to backtrack over predicates that do I/O, we have to state that we’re content for \texttt{main} to stick to the first solution it finds, whichever one that happens to be.

Now to outline how \texttt{pick} works; here is its definition again:

\begin{verbatim}
:- pred pick(list(int)::in, int::out, list(int)::out) is nondet.
pick([X | Xs], X, Xs). % Clause 1.
pick([X | Xs], Y, [X | Zs]) :- pick(Xs, Y, Zs). % Clause 2.
\end{verbatim}

The first argument is an input, the second and third are outputs. The \texttt{nondet} determinism category means that \texttt{pick} can have any number of solutions, including zero, depending upon its input. Clause 1 says that we can pick an item from a list whose first (head) item is \texttt{X} and whose tail is \texttt{Xs} by returning \texttt{X} as the chosen item and \texttt{Xs} as the remainder. Clause 2 says that we can also pick an item from our list by choosing some \texttt{Y} from \texttt{Xs}, leaving \texttt{Zs}, and returning \texttt{Y} as the chosen item and the list \texttt{[X | Zs]} as the remainder.

For example, The goal \texttt{pick([1, 2, 3], X, Xs)} has three possible solutions:

\begin{verbatim}
pick([1, 2, 3], 1, [2, 3]) % By clause 1.
pick([1, 2, 3], 2, [1, 3]) % By clause 2 because
  pick([2, 3], 2, [3]) % by clause 1.
pick([1, 2, 3], 3, [1, 2]) % By clause 2 because
  pick([2, 3], 3, [2]) % by clause 2 because
    pick([3], 3, []) % by clause 1.
\end{verbatim}

Each of these solutions will be enumerated by Mercury on backtracking. (Note that the goal \texttt{pick([], X, Xs)} has no solution because there are no clauses matching the empty list [] as an input.)

Don’t worry if much of this seems confusing: once some facility with thinking declaratively (i.e., “what” rather than “how”) is acquired, one can look
at predicates like \texttt{pick} and complicated goals like the condition of this \texttt{if-then-else} and immediately understand what is meant. The thing to carry away from this example is the ease with which Mercury allows us to describe a solution to a complex search problem. We have done very little more than write a logical specification of the problem using Mercury syntax, all the tedious operational details are handled for us by the compiler! Elegance of this sort is one of the key things that makes Mercury such an attractive programming language.

Anyway, after all that hard work, let’s just prove that all this magic actually works:

\begin{verbatim}
$ mmc --make crypt
Making Mercury/int3s/crypt.int3
Making Mercury/cs/crypt.c
Making Mercury/os/crypt.o
Making crypt
$ ./crypt
DOG + ANT = CAT
420 + 531 = 951
\end{verbatim}

Success!

\textbf{Points to remember}

- Nondeterministic predicates such as \texttt{pick} can have multiple solutions for a given set of inputs. Mercury handles this by backtracking to the most recent choice point when a later goal fails.

- As far as possible, try to think declaratively rather than operationally: focus on what it is that is being computed rather than how the computation should proceed. Leave as much of that sort of detail to the compiler as you can.

- Unification goals are quite versatile: they can denote constructions, deconstructions, and equality tests.
Chapter 2

The Mercury type system

In this chapter we describe the primitive (i.e., built-in) Mercury types, and how to define and use new types. The style of this chapter is a little dry, so we advise the reader to skim through it the first time around, and then dip back into it for reference as occasion demands.

Mercury uses an expressive, statically checked type system similar to that of ML and Haskell. The type system is expressive in the sense that the compiler can always infer the type held in a particular variable; apart from one exception, which is described below, type casts of the kind found in Java and C programs are unnecessary. Static type checking means that any possible type violation in a program is detected at compile-time rather than at run-time. Many common programming errors are, therefore, simply inexpressible in Mercury. A handy secondary benefit of this approach to typing is that the compiler can generate highly optimized code.

Mercury also supports dynamic typing and type reflection for the rare cases where such things are necessary. Dynamic typing is supported by a universal type that can conceal values of any type at all; however, extracting a concealed value does require a checked run-time type cast operation. Type reflection allows a program to examine the structure of values and their types. By way of example, a generic function to compute hash codes for values of any type depends upon type reflection.
Important!

Mercury is a purely declarative language. This means that values are immutable: there is no destructive assignment. Every computation constructs a new value for each output. The compiler may well generate code that uses destructive assignment (e.g., for efficient array updates), but such things are not directly available to the Mercury programmer.

2.1 The primitive types: int, float, string, and char

Note! The underlying representation of the primitive types depends upon the compiler target (C, Java, .Net etc.)

int

Fixed precision integers are represented by the int type. Syntactically, an int is a sequence of digits, optionally preceded by a minus sign (there is no unary plus). The sequence of digits may be decimal, hexadecimal (both uppercase and lowercase are allowed, preceded by 0x), octal (if preceded by 0o), or binary (if preceded by 0b).

Examples: decimal -123, 0, 42; hexadecimal -0x7B, 0x0, 0x2a; octal -0o173, 0o0, 0o52; binary -0b1111011, 0b0, 0b101010.

The sequence 0’x denotes the character code for the character x. For example, on an ASCII system 0’a, 0’b, and 0’c denote 97, 98, and 99 respectively.

The int standard library module must be imported to use any of the primitive int operations.

float

Floating point numbers are represented by the float type, which corresponds to the C double type. Syntactically, a float is a decimal floating
2.1. **THE PRIMITIVE TYPES: INT, FLOAT, STRING, AND CHAR**

point number (the decimal point is required), optionally preceded by a minus sign, optionally followed by an integer exponent.

These are all equivalent: 1.414, 1414e-3, .1414e1, 0.01414e2; either e or E is acceptable as the exponent separator.

The **float** standard library module must be imported to use any of the primitive **float** operations. Constants such as **pi** and **e** and more complex floating point operations, such as the trigonometric functions, are defined in the **math** standard library module.

**string**

A string constant is a sequence of characters enclosed in double quotes.

Examples: " ", "Hello, World!\n", "'Lawks!'" I declared."

Certain characters have special syntax referred known as **character escapes**:

- `\"` double quote
- `\\` backslash
- `\` single quote
- `\a` alert ("beep")
- `\b` backspace
- `\r` carriage return
- `\f` form-feed
- `\t` tab
- `\n` newline
- `\v` vertical tab

Characters can also be specified by character code using the sequence `\xhh\`, where hh is a hexadecimal number, or `\ooo\`, where ooo is an octal number. the ASCII character 'A', for example, can also be written as `\x41\` or `\101\`.

**Note!** For arcane reasons, it is a Very Bad Idea to include the **NUL** character, `\x00\`, in strings.

A backslash at the end of a line is ignored in string constants. Thus

```
"abc\ndef"
```

is equivalent to just "abcdef". Otherwise, literal newlines may appear in a string constant:

```
"pqr
xyz"
```

is equivalent to "pqr\nxyz".

The **string** standard library module must be imported to use any of the
primitive **string** operations. In particular it defines `++`, the string concatenation function — for example, "foo" ++ "bar" = "foobar".

**char**

Character constants are represented using the `char` type and, syntactically, are single characters (or character escapes or character codes as described above) enclosed in single quotes. Characters that could be interpreted as infix operators, such as `+` and `*`, should be further enclosed in parentheses.

Examples: 'A', '\x41\', '\101\', '\', '\n', ('+', ('*')).

**XXX Are we ASCII specific?**

The `char` standard library module must be imported to use any of the primitive operations on `chars`.

### 2.2 Tuples

A tuple is a fixed size vector of values. Syntactically, a tuple **type** is a comma separated sequence of type names enclosed in braces, whereas a tuple **value** is a comma separated sequence of values enclosed in braces.

Examples: `{111, 'b'}` is a value of type `{int, char}; `{1.2, 3, "456"}` is a value of type `{float, int, string}; `{"a", "little", "contrived"}` is a value of type `{string, {string, string}}.

**Note!** Unlike lists (described below), tuples are constructed and deconstructed as complete entities. There is no such thing as the head or tail of a tuple. **XXX Should we include this note at all? Or move the whole tuples section after lists?**

Tuples are occasionally useful for aggregating small numbers of different types. More often than not it is better style to use a discriminated union type.
2.3 Lists

A list is a linear sequence of values sharing the same type. Syntactically, a list is a comma separated sequence of values enclosed in brackets.

Examples: [] denotes the empty list, regardless of the list type; [1, 2, 3] is a value of type list(int); ['a', 'b', 'c', 'd'] is a value of type list(char); [[1], [2, 3], [4]] is a value of type list(list(int)).

It is an error to mix values of different types inside a list because there is no way to give a type to values such as [1, "2", 3.4].

The list standard library module defines the list type and a wealth of list operations. If you forget to import the list module then the compiler will issue warnings like this:

foo.m:031: error: undefined symbol ‘[]/2’
foo.m:031: (the module ‘list’ has not been imported).

Lists are actually composed of two kinds of building block: [] (the empty list) and [ | ] (the non-empty list constructor, pronounced “cons”). If X is an int, say, and Xs a list(int), then [ | ](X, Xs) is a list(int) whose head (first member) is X and whose tail (list of trailing members) is Xs.

Lists are so common that special syntactic sugar exists for them: [X | Xs] is shorthand for [ | ](X, Xs); [X, Y, Z | Ws] is shorthand for [X | [Y | [Z | Ws]]]; and [X, Y, Z] is shorthand for [X, Y, Z | []].

2.4 Discriminated union types

Discriminated unions allow the definition of new, structured types. This example shows a representation of playing cards using discriminated union types:

:- type playing_card ---+ card(rank, suit) ; joker.
:- type rank ---+ ace ; two ; three ; four
; five ; six ; seven ; eight
; nine ; ten ; jack ; queen ; king.
:- type suit ---+ clubs ; diamonds ; hearts ; spades.
The data constructors defining the values of the discriminated union types appear to the right of the arrows: suit has four possible values, rank thirteen, and playing_card fifty three (fifty two possible card values from card(ace, clubs), card(two, clubs), all the way up to card(king, spades), plus the joker option).

An exhaustive switch on a discriminated union type must test for every possible top-level data constructor. That is, an exhaustive switch on a playing_card value need only test for card(_, _) and joker rather than every specific card instance.

(The term “discriminated union” is used because a type denotes a union of sets of possible values, each of which is distinguished by its data constructor.)

Data constructors with named fields

The fields of a data constructor can be named:

```prolog
:- type bank_account ---> account( name :: string, account_no :: int, funds :: float ).
```

We can use field names to access fields directly without having to first deconstruct a bank_account value. That is, rather than writing

```prolog
BankAcct = account(Name, AcctNo, Funds),
( if Funds >= RequestedSum then
  ... debit RequestedSum from BankAcct ...
else
  ... reject debit request ...
)
```

we can write

```prolog
( if BankAcct^funds >= RequestedSum then
  ... debit RequestedSum from BankAcct ...
else
  ... reject debit request ...
)
```
2.4. **DISCRIMINATED UNION TYPES**

The Mercury compiler automatically generates a *field access function* for every named field in a data constructor. The `bank_account` field access functions would be defined like this:

\[
\begin{align*}
\text{account}(A, _, _) \cdot \text{name} &= A. \\
\text{account}(_, B, _) \cdot \text{account_no} &= B. \\
\text{account}(_, _, C) \cdot \text{funds} &= C.
\end{align*}
\]

Field access functions to “update” named fields are also generated:

\[
\begin{align*}
( \text{if } \text{BankAcct} \cdot \text{funds} \geq \text{RequestedSum} \text{ then } \\
\text{NewBankAcct} &= (\text{BankAcct} \cdot \text{funds} := \text{BankAcct} \cdot \text{funds} - \text{RequestedSum}) \\
\text{else } \\
\text{... reject debit request ...})
\end{align*}
\]

The expression \((\text{BankAcct} \cdot \text{funds} := X)\) returns a value identical to \(\text{BankAcct}\) except the \text{funds} field will contain \(X\). The `bank_account` field access update functions would be defined like this:

\[
\begin{align*}
( \text{account}(_, B, C) \cdot \text{name} := A ) &= \text{account}(A, B, C). \\
( \text{account}(A, _, C) \cdot \text{account_no} := B ) &= \text{account}(A, B, C). \\
( \text{account}(A, B, _) \cdot \text{funds} := C ) &= \text{account}(A, B, C).
\end{align*}
\]

Note that you do not have to name every field of a data constructor; unnamed fields can only be “read” or “updated” by explicitly deconstructing or constructing the entire data constructor value.

*You are not allowed to use the same field name in different types defined in the same module.* This is an error:

\[
\begin{align*}
:- \text{type cat} &\longrightarrow \text{cat(name :: string)}. \\
:- \text{type dog} &\longrightarrow \text{dog(name :: string)}.
\end{align*}
\]

Instead use distinct field names, such as `cat_name` and `dog_name`, or use a single type with two data constructors (the same field name can be used in different data constructors of the same type.)

*A field access may fail if a data type has more than one data constructor.* For example, given
CHAPTER 2. THE MERCURY TYPE SYSTEM

:- type playing_card ---> card(card_rank :: rank, card_suit :: suit) ; joker.

A goal featuring the expression Card^card_rank will fail if Card happens to be a joker.

Field accesses can be chained together.

:- type employee ---> employee(id :: int, contact :: contact_details).
:- type contact_details ---> contact_details(address :: string, phone :: int).

If Employee contains a value of type employee then the expression Employee^contact^address is the address field of the contact field of the employee constructor.

Nested fields can be updated. The expression Employee^contact^address := NewAddr denotes a copy of Employee with the address field of the contact field of the employee data constructor updated to hold NewAddr.

Parentheses can change the meaning of an update expression:
(Employee^contact)^address := NewAddr denotes an updated copy of the contact field of the employee data constructor (i.e., the type of this expression is contact_details, not employee.)

One final remark: it is also possible to explicitly define field access functions, for instance for “virtual fields” that are computed rather than stored in a data constructor or for update functions that perform sanity checks on their arguments. User defined field access functions are described fully in Chapter XXX.

2.5 Polymorphic types

Polymorphic types are types parameterised by type variables. A polymorphic binary tree type carrying values at the branches could be defined like this:

:- type tree(T) ---> leaf ; branch(tree(T), T, tree(T)).

This is just a discriminated union type with a type argument, T. T can match any type at all, so tree(int), tree(string), tree(list(char)) are all refinements of tree(T).
Examples: \(\text{branch}(\text{branch}(	ext{leaf}, 1, \text{leaf}), 2, \text{branch}(	ext{leaf}, 3, \text{leaf}))\) is a value of type \(\text{tree}(\text{int})\); \(\text{branch}(\text{leaf}, \{\text{a}', 65\}, \text{branch}(\text{leaf}, \{\text{b}', 66\}, \text{leaf}))\) is a value of type \(\text{tree}({\text{char}, \text{int}})\); and \(\text{leaf}\) is a value of every \(\text{tree}\) type.

The canonical example of a polymorphic data type is the \(\text{list}\) type defined in the \(\text{list}\) standard library module:

\[
:\text{- type list}(T) \rightarrow [] ; [T \mid \text{list}(T)].
\]

The \(\text{maybe}\) type defined in the \(\text{std\_util}\) standard library module is another useful polymorphic type:

\[
:\text{- type maybe}(T) \rightarrow \text{no} ; \text{yes}(T).
\]

This type is commonly used to represent optional values (had the database community known about \(\text{maybe}\) types they never would have invented \text{NULL}s and wrecked the relational model...)

Once one has polymorphic types, it is natural to want to define polymorphic predicates and functions. The \(\text{list}\) module \(\text{length}\) function works for lists of every type thanks to the type argument in its signature (it is just convention that we have reused the name \(T\) here — any variable name would do):

\[
:\text{- func length}(\text{list}(T)) = \text{int}.
\]

\[
\begin{align*}
\text{length}([]) & = 0. \\
\text{length}([_ \mid Xs]) & = 1 + \text{length}(Xs).
\end{align*}
\]

The first clause defines the length of the empty list to be 0; the second clause defines the length of a non-empty list \([_ \mid Xs]\) to be 1 for the head (the underscore says we don’t care what that happens to be) plus the length of the tail, \(Xs\).

Here is an example of a polymorphic predicate to decide whether a given value resides in an ordered binary tree of the type we defined at the start of this section (by ordered we mean that smaller values appear to the left of larger values in the tree):
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:- pred search(tree(T)::in, T::in) is semidet.

search(branch(L, X, R), Y) :-
    O = ordering(X, Y),
    ( O = (\textless), search(R, Y)
    ; O = (=)
    ; O = (\textgreater), search(L, Y)
    ).

The ordering function is built-in to Mercury and compares any two values of the same type, returning a result of type comparison_result:

:- type comparison_result --\rightarrow (\textless); (=); (\textgreater).

As you can see, data constructor names don’t necessarily have to be alphanumerical. These data constructor names must appear in parentheses to stop the Mercury parser from interpreting them as infix operators. This is also an example of overloading, where the same name may be used for more than one purpose provided there is no ambiguity.

After calling ordering, search switches on O to decide what to do next: if the value at the current branch, X, is less than the value we are searching for, Y, then search should proceed down the right subtree, R. If X = Y then the search terminates successfully. Otherwise, X is greater than Y and the search should proceed down the left subtree, L. The absence of a clause for leaf values means that any search that reaches a leaf will fail.

2.6 Equivalence types

Readability is often improved by giving simple names to complex types or by using more meaningful names for a specific uses of general types:

:- type height == float. % In metres.
:- type radius == float. % In metres.
:- type volume == float. % In cubic metres.

:- func volume_of_cylinder(height, radius) = volume.
:- func volume_of_sphere(radius) = volume.
2.7. **ABSTRACT TYPES**

XXX *Move this stuff about comments to chapter 1.* (The % sign introduces a comment, which extends to the end of the line.) Here we define the types `height`, `radius` and `volume` to be equivalent to (i.e., interchangeable with) type `float`. We could have just declared `volume_of_cylinder` using

\[-\text{func volume_of_cylinder}(\text{float}, \text{float}) = \text{float}.\]

but then we would be morally obliged to include a comment explaining which arguments correspond to which measurements.

Equivalence types can also be parameterized. For example:

\[-\text{func volume_of_cylinder}(\text{float}, \text{float}) = \text{float}.\]

\[-\text{type dictionary}(\text{Key}, \text{Value}) \equiv \text{list}(\{\text{Key}, \text{Value}\}).\]

\[-\text{pred search(dictionary}(\text{Key}, \text{Value}):\text{in}, \text{Key}:\text{in}, \text{Value}:\text{out}) \text{ is semidet.}\]

\[-\text{search}([\{K, V\} | \text{Dict}], \text{Key}, \text{Value}) :-\]
\[-\quad (\text{if Key} = K \text{ then Value} = V \text{ else search(Dict, Key, Value)}) \text{.}\]

\[-\text{set}((\text{Dict}, \text{Key}, \text{Value}) \text{ returns an updated version of Dict}\]
\[-\quad \text{set(Dict, Key, Value)} \text{ associates Key with Value.}\]
\[-\text{set}((\text{Dict}, \text{Key}, \text{Value}) = ([\{\text{Key}, \text{Value}\}] | \text{Dict}).\]

### 2.7 Abstract types

It is virtually always a Bad Idea to reveal implementation detail to the user of a module. Mercury ensures that predicate and function definitions are private to a module because they cannot appear in the interface section of a module. Abstract types allow the same kind of information hiding for types. An abstract type is one that is declared in the interface section of a module, but defined in the implementation section.

Here’s how we would use abstract types if we wanted to export the `dictionary` type defined above:
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```prolog
:- module dictionary.
:- interface.

:- type dictionary(Key, Value).

:- pred search(dictionary(Key, Value)::in, Key::in, Value::out) is semidet.
:- func set(dictionary(Key, Value), Key, Value) = dictionary(Key, Value).

:- implementation.
:- import_module list.

:- type dictionary(Key, Value) == list({Key, Value}).

search([{K, V} | Dict], Key, Value) :-
  ( if Key = K then Value = V else search(Dict, Key, Value) ).

set(Dict, Key, Value) = [{Key, Value} | Dict].
```

Observe the type declaration in the interface section: it gives the name of the type and its arguments, but nothing else. Further down, in the implementation section, we give a definition for dictionary. At some later point we may reimplement dictionary as an ordered list or binary tree or some other more efficient structure. Such a change would not affect the interface of the dictionary module (because dictionary is an abstract type), so no changes would be required by users of the module.

### 2.8 Higher order types

Mercury considers functions and predicates to be values just as much as it does ints, strings and lists. Consider the higher order `map` function (as defined in the list standard library module) which takes a function from T1 values to T2 values, a list of T1 values, and returns a list of T2 values:

```prolog
:- func map(func(T1) = T2, list(T1)) = list(T2).

map(_, []) = [].
map(F, [X | Xs]) = [F(X) | map(F, Xs)].
```

The first clause says that mapping over the empty list returns the empty list. The second clause says that mapping the function F over the non-empty
list \([X \mid Xs]\) is the list whose head is \(F(X)\) (i.e., the result of applying \(F\) to \(X\)) and whose tail is the result of mapping \(F\) over \(Xs\).

The thing to observe here is the argument type \(\text{func}(T1) = T2\), which illustrates the syntax for function types.

This next example illustrates the syntax for predicate types (this predicate is also defined in the \textit{list} standard library module):

\[
: - \text{pred} \quad \text{filter}(\text{pred}(T), \text{list}(T), \text{list}(T), \text{list}(T)) .
: - \text{mode} \quad \text{filter}(\text{in}(\text{pred}(\text{in}) \text{ is semidet}), \text{in}, \text{out}, \text{out}) \text{ is det} .
\]

\[
\text{filter}(\_, [], [], []). \\
\text{filter}(P, [X \mid Xs], Ys, Zs) :- \\
\quad \text{filter}(P, Xs, Ys0, Zs0), \\
\quad \quad ( \text{if } P(X) \text{ then } Ys = [X \mid Ys0], \ Zs = Zs0 \\
\quad \quad \text{else } Ys = Ys0, \ Zs = [X \mid Zs0] \\
\).
\]

the goal \(\text{filter}(P, As, Bs, Cs)\) unifies \(Bs\) with the list of members of \(As\) that satisfy \(P\) and unifies \(Cs\) with the list of members of \(As\) that don’t. The first clause says that filtering the empty list yields two empty lists. The second clause says that filtering \([X \mid Xs]\) through the predicate \(P\) is the result of filtering \(Xs\) through \(P\) and adding \(X\) to the first result if \(P(X)\) (i.e., if \(P\) succeeds given \(X\)), or adding \(X\) to the second result if it doesn’t.

The first new thing here is the separation of \textit{type} information from \textit{mode} information in the declarations for \textit{filter}. Mercury requires a separate \texttt{mode} declaration if you do not supply mode and determinism details in the \texttt{pred} declaration. The two declarations could be combined thus

\[
: - \text{pred} \quad \text{filter}(\text{pred}(T)::\text{in}(\text{pred} \text{ (in) is semidet}), \text{list}(T)::\text{in}, \text{list}(T)::\text{out}, \text{list}(T)::\text{out}) \text{ is det} .
\]

However, having separate \texttt{pred} and \texttt{mode} declarations highlights that the \textit{type} of \textit{filter}’s higher order argument is written \texttt{pred}(T). (Separate \texttt{mode} declarations are examined in more detail in Chapter \textit{XXX}.)

XXX Do I need to explain the parameterised \texttt{in} \texttt{mode}?}

In general, higher order programming with predicates is more complicated than with functions because of the need to also specify the modes for the higher order arguments.
2.9  \texttt{univ}, the universal type

The \texttt{univ} type provides support for dynamically typed programming. Mercury's type system is so expressive that \texttt{univ} is hardly ever necessary. However, should you require it, here is (a slightly abridged version of) the interface to \texttt{univ} as defined in the \texttt{univ} standard library module:

\begin{verbatim}
:- type univ.
:- func univ(T) = univ.
:- pred univ_to_type(univ::in, T::out) is semidet.
\end{verbatim}

The \texttt{univ} function turns an argument of any type into a \texttt{univ} value (this is an example of overloading a name for a type and a function). The \texttt{univ_to_type} predicate turns a \texttt{univ} value into a value of type \texttt{T} (what \texttt{T} denotes depends upon the context of the call to \texttt{univ_to_type}) if that is the type of value contained in the \texttt{univ}, and fails otherwise.

We said earlier that a value like \([1, "2", 3.4]\) would be rejected because it cannot be given a type. This is true, but we can achieve much the same end by writing \([\texttt{univ}(1), \texttt{univ}("2"), \texttt{univ}(3.4)]\), which does have a type, \texttt{list(univ)}.

To illustrate the use of \texttt{univ_to_type}, here is a program to print out \texttt{univ}s:

\begin{verbatim}
:- module print_univs.
:- interface.
:- import_module io.

:- pred main(io::di, io::uo) is det.

:- implementation.
:- import_module list, string, univ.

main(!IO) :-
    print_univ(univ(1), !IO),
    print_univ(univ("2"), !IO),
    print_univ(univ(3.4), !IO),
    print_univ(univ([5, 6, 7]), !IO).

:- pred print_univ(univ::in, io::di, io::uo) is det.
\end{verbatim}
2.10. USEFUL TYPES DEFINED IN THE MERCURY STANDARD LIBRARY

print_univ(U, !IO) :-
    ( if univ_to_type(U, C) then io.format("a char, %c\n", [c(C)], !IO)
      else if univ_to_type(U, S) then io.format("a string, \’\’%s\’\’\n", [s(S)], !IO)
      else if univ_to_type(U, I) then io.format("an int, %d\n", [i(I)], !IO)
      else if univ_to_type(U, F) then io.format("a float, %f\n", [f(F)], !IO)
      else io.format("no idea...\n", [], !IO)
    ).

Compiling and running this program, we get

> mmc --make print_univs
Making Mercury/int3s/print_univs.int3
Making Mercury/cs/print_univs.c
Making Mercury/os/print_univs.o
Making print_univs
> ./print_univs
an int, 1
a string, ‘’2’’
a float, 3.400000
no idea...

So how does univ_to_type know that C is a char, S a string, and so forth? The answer is the compiler automatically infers these types from context: the argument of a c data constructor in an io.format argument list must be a char; the argument of an s data constructor must be a string; similarly I must be an int and F a float. Information about each result type is supplied to the univ_to_type calls via extra arguments inserted by the Mercury compiler.

Full details of the run-time type information (RTTI) scheme are beyond the scope of this book. The interested reader is referred to the documentation for the std_util module in the Mercury Library Reference Manual and the relevant parts of the Mercury Reference Manual.

2.10 Useful types defined in the Mercury standard library

The Mercury standard library defines many useful types, the most common of which are examined in more detail in Chapter XXX. These include
bool for Boolean values, digraph for directed graph processing, list for stacks and sequences, map for dictionaries, queue for first-in first-out (FIFO) queues, pqueue for priority queues, random for random numbers, and set for sets. Many more types are defined in the standard library, but the above suffice for the majority of data structures.

2.11 More advanced types...

Mercury’s type system includes two more advanced aspects each with a separate chapter. Chapter XXX describes type classes and existentially quantified types which exist to support object oriented programming styles. Chapter XXX describes types with user-defined equality and comparison relations; these are so-called non-canonical types in which a given semantic value may be represented in more than one way.

XXX I haven’t really mentioned type inference, nor have I mentioned explicit type qualification of local vars.
Chapter 3

The Mercury mode system

Mercury programs are really just logical formulae written down using a particular syntax. The Mercury compiler, however, needs extra information to turn these logical formulae into something the computer can execute. Specifically, it needs to know which arguments of a predicate can be inputs and which outputs. This information is conveyed using argument modes and determinism categories.

This chapter explains the mode system and how it is used.

3.1 Predicates and procedures

Every predicate must have a declaration specifying which arguments can be inputs and which outputs. Consider the following implementation of a telephone directory in which phone numbers can be looked-up by name:

```mercury
:- pred phone(string::in, int::out) is semidet.

phone("Ian", 66532).
phone("Julien", 66532).
phone("Peter", 66540).
phone("Ralph", 66532).
phone("Zoltan", 66514).
```

CHAPTER 3. THE MERCURY MODE SYSTEM

The style of \texttt{pred} declaration used here is called a \texttt{pred-mode} declaration and is syntactic sugar for two separate declarations:

\begin{verbatim}
:- pred phone(string, int).
:- mode phone(in, out) is semidet.
\end{verbatim}

The plain \texttt{pred} declaration tells us the argument types; the \texttt{mode} declaration tells us the \textit{argument modes} (\texttt{in} or \texttt{out}) and the corresponding \textit{determinism category} (\texttt{semidet}).

It is natural to want to call some predicates in more than one way — if we want to use \texttt{phone} to perform “reverse look-ups”, for instance. In such cases more than one \texttt{mode} declaration is necessary (\texttt{pred-mode} shorthand can only be used for singly-moded predicates).

To allow reverse look-ups with \texttt{phone} all that is necessary is to (a) use separate \texttt{pred} and \texttt{mode} declarations and (b) add an extra \texttt{mode} declaration:

\begin{verbatim}
:- pred phone(string, int).
:- mode phone(in, out) is semidet.
:- mode phone(out, in) is nondet.
\end{verbatim}

The first \texttt{mode} declaration

\begin{verbatim}
:- mode phone(in, out) is semidet.
\end{verbatim}

says that if we call \texttt{phone} giving its first argument as an input and taking its second as an output, then the result is semideterministic: every name in the directory appears exactly once, but not all names are listed. The goal \texttt{phone("Harald", HaraldsNum)} will obviously fail, but \texttt{phone("Ralph", RalphsNum)} will succeed unifying \texttt{RalphsNum} with 66532.

The second \texttt{mode} declaration

\begin{verbatim}
:- mode phone(out, in) is nondet.
\end{verbatim}

says that if we call \texttt{phone} with its second argument given as input and take its first argument as output, then the result is nondeterministic: it can fail because the goal \texttt{phone(Person, 12345)} will fail, but the goal
3.2. **THE DETERMINISM CATEGORIES**

`phone(Person, 66532)` has no less than three possible solutions — `Person = "Ian"`, `Person = "Julien"`, and `Person = "Ralph"` — each of which will be computed on backtracking.

**Note!** Each mode declaration specifies a *procedure* to be derived from the predicate definition. The compiler generates code separately for each procedure of a predicate, reordering goals to ensure that every variable is instantiated (e.g., by some earlier unification or call) before it is used in an input in a unification or call. Mode information is also used to decide whether a unification with a data constructor is a construction or deconstruction.

The compiler verifies that the determinism category for a procedure properly describes the behaviour of the procedure. The compiler will issue an error if a procedure can fail or have multiple solutions when its declared determinism category says otherwise. Moreover, the compiler will report an error if a deterministic switch is incomplete, telling you which cases have been missed (Prolog programmers dream of having error detection like this...)

**Implied modes**

An *implied mode* is one where an output argument is supplied as an input in a procedure call. Consider the goal `phone("Ralph", 66540)`. The compiler gets around this situation by placing a new, temporary variable in the output position and then adding a unification goal, giving `phone("Ralph", Tmp), Tmp = 66540`.

### 3.2 The determinism categories

A determinism category tells us whether a particular procedure can fail and whether it may have more than one solution:

<table>
<thead>
<tr>
<th>Determinism category</th>
<th>Number of solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>det</td>
<td>1</td>
</tr>
<tr>
<td>semidet</td>
<td>≤ 1</td>
</tr>
<tr>
<td>multi</td>
<td>≥ 1</td>
</tr>
<tr>
<td>nondet</td>
<td>≥ 0</td>
</tr>
<tr>
<td>failure</td>
<td>0</td>
</tr>
</tbody>
</table>

There are three other determinism categories that are only occasionally needed: **erroneous**, which is used for predicates that only terminate by throwing an exception (exceptions are described in Chapter **XXX**), and...
cc_multi and cc_nondet which are used for committed-choice nondeterminism (see Chapter XXX).

Some examples

:- pred square(int::in, int::out) is det.
  square(X, X * X).

square is det: it cannot fail and every input has a single solution for the output.

:- pred absolute_square_root(float::in, float::out) is semidet
  absolute_square_root(X, AbsSqrtX) :-
    X >= 0.0,
    AbsSqrtX = math.sqrt(X).

absolute_square_root is semidet: it fails for negative inputs while non-negative inputs each have a single solution.

:- pred small_prime(int::out) is multi.
  small_prime(2).
  small_prime(3).
  small_prime(5).
  small_prime(7).

small_prime is multi: it cannot fail and it has more than one solution.

:- pred small_prime_factor(int::in, int::out) is nondet.
  small_prime_factor(X, P) :-
    small_prime(P),
    X mod P = 0.

small_prime_factor is nondet: small_prime_factor(11, A), for instance, will fail, but small_prime_factor(6, A) has solutions A = 2 and A = 3.

Finally, the built-in predicate false, which takes no arguments, has determinism failure: it never succeeds. The opposite of false is the built-in
3.3. DETERMINISM

Note! The determinism category of a goal with no output arguments is either det, semidet, or failure. Consider the following:

\[-\text{pred has_small_prime_factor(int::in) is semidet.}\]
\[-\text{has_small_prime_factor}(X) :\]
\[-\text{small_prime}(P),\]
\[-X \text{ mod } P = 0.\]

Because there are no outputs, Mercury ensures that the goal 
\text{has_small_prime_factor}(15), say, will not succeed more than once, even 
though \text{small_prime}(P) has two solutions, \(P = 3\) and \(P = 5\), satisfying 
\(15 \text{ mod } P = 0\).

3.3 Determinism

These rules specify how determinism categories for compound goals are derived (with a little experience this quickly becomes second nature). The determinism category of a goal is derived from the instantiation state of its arguments at the time the goal is executed.

Note! Remember that the compiler reorders the goals in a predicate separately for each mode declaration for the predicate. A running program does not make decisions about which procedures should be executed when calling predicates; this is decided in advance by the Mercury compiler.

Unifications

Whether a unification is a construction, deconstruction, assignment or equality test depends upon which variables are instantiated and which are not at the time the unification is executed.

A unification \(X = \text{data_ctor}(Y_1, Y_2, Y_3)\) is a construction if \(Y_1, Y_2,\) and \(Y_3\) are initially instantiated and \(X\) is not. Constructions are always det. Afterwards, \(X\) will be instantiated.
CHAPTER 3. THE MERCURY MODE SYSTEM

A unification $X = \text{data ctor}(Y_1, Y_2, Y_3)$ is a deconstruction if $X$ is initially instantiated. Afterwards, $Y_1$, $Y_2$, and $Y_3$ will be instantiated. Deconstructions are almost always semidet (in certain circumstances a deconstruction may have determinism category det if it is guaranteed to succeed or failure if it is guaranteed to fail).

A unification $X = Y$ is an assignment if precisely one of $X$ or $Y$ is initially instantiated. Afterwards, both variables will be instantiated. Assignments are always det.

A unification $X = Y$ is an equality test if both $X$ and $Y$ are initially instantiated. Equality tests are always semidet.

Procedure calls

For a predicate call $p(X_1, X_2, X_3)$, which procedure of $p$ is executed depends upon which mode declaration for predicate $p$ best matches the instantiation states of $X_1$, $X_2$, and $X_3$ at the time the call is executed. The determinism category of the goal is that of the called procedure, adjusted for any implied modes (i.e., extra unifications added because some output arguments of the procedure are already instantiated at the time of the call).

For example, the goal $\text{phone}("Zoltan", \text{ZoltansNumber})$ is compiled as a call to the (in, out) is det procedure of $\text{phone}$. The goal $\text{phone}(\text{Person}, 66540)$ is compiled as a call to the (out, in) is nondet procedure of $\text{phone}$. The goal $\text{phone}("Ralph", 66532)$ requires an implied mode and may be compiled either as $\text{phone}("Ralph", \text{Tmp}), \text{Tmp} = 66532$ or $\text{phone}(\text{Tmp}, 66532), \text{Tmp} = "Ralph"$, both of which are semidet.

Conjunction

A sequence of goals separated by commas, $G_1, G_2, G_3, \ldots$, is called a conjunction. The commas are pronounced “and” and each subgoal is called a conjunct.

A conjunction can fail if any conjunct can fail.

A conjunction can succeed if every conjunct can succeed.

A conjunction can have multiple solutions if it can succeed and one or more
conjuncts have multiple solutions.

**Note!** These rules are a conservative (i.e., safe) approximation. For example, the compiler will conclude that conjunction `small_prime(X), X = 4` is **semidet**, even though we can see that this goal has to fail.

### Disjunction

A sequence of goals separated by semicolons, `(G1 ; G2 ; G3 ; ...)`, is called a *disjunction*. The semicolons are pronounced “or” and each subgoal is called a *disjunct*.

A disjunction can succeed if any disjunct can succeed.

A disjunction can have multiple solutions if more than one disjunct can succeed or one or more disjuncts can have multiple solutions.

**Note!** Switches are a special case. A switch is a disjunction that deconstructs a particular variable against a different data constructor in each disjunct. If, apart from the deconstructions, every disjunct is **det**, then the switch is **det** if the set of deconstructions is exhaustive and **semidet** if not.

For example, even though both `p` and `q` (below) define switches on `X`, `p` is **det** because its switch is exhaustive, whereas `q` is **semidet** because its switch is not:

```prolog
:- type ott ---> one ; two ; three.

:- pred p(ott::in, int::out) is det.
p(X, Y) :- ( X = one, Y = 1 ; X = two, Y = 2 ; X = three, Y = 3 ).

:- pred q(ott::in, int::out) is semidet.
q(X, Y) :- ( X = one, Y = 1 ; X = three, Y = 3 ).
```

**Note!** Disjunction binds less tightly than conjunction:

( `G11, G12 , G13 ; G21 ; G31, G32` )

is equivalent to

( `(G11, G12, G13) ; G21 ; (G31, G32)` ).

**Note!** A definition spanning multiple clauses is equivalent to a definition using a single clause containing a disjunction. That is
p(one, 1).
p(two, 2).
p(three, 3).

is semantically and operationally identical to

p(X, Y) :- ( X = one, Y = 1 ; X = two, Y = 2 ; X = three, Y = 3 ).

**Note!** If any disjunct instantiates a variable that is used outside the disjunction, then every disjunct in the disjunction must also instantiate that variable. That is, the Mercury compiler will report a mode error if a program contains a disjunction that instantiates X in some disjuncts, but not others, and X is also needed outside the disjunction. For instance, the following is illegal because Y, which appears outside the disjunction, is instantiated in the first and second disjuncts, but not the third:

:- pred p(number::in, int::out) is det.
p(X, Y) :- ( X = one, Y = 1 ; X = two, Y = 2 ; X = three ).

**Negation**

A goal (not G) is called the *negation* of G. The negation fails if G succeeds, and vice versa. The negation succeeds if G fails and fails if G succeeds.

**Note!** G is said to occur inside a *negated context* and is not allowed to instantiate variables that also occur outside the negation.

**Note!** Negation binds more tightly than conjunction, hence not G1, G2, ... is equivalent to (not G1), G2, .... To negate a conjunction, put the conjunction in parentheses: not (G1, G2, ...)  

**Note!** X \(\neq\) Y is syntactic sugar for not (X = Y).

**If-then-else goals**

The declarative semantics for a goal ( if Gc then Gt else Ge ) are identical to those of ( Gc, Gt ; (not Gc), Ge ). The operational semantics
are more efficient, though: if there are no solutions to $G_c$, the program immediately executes $G_e$.

If any of $G_c$, $G_t$, or $G_e$ can fail then the if-then-else can fail.

If any of $G_c$, $G_t$, or $G_e$ can have multiple solutions then the if-then-else can have multiple solutions.

**Note!** $G_c$ is not allowed to instantiate variables that are used outside the if-then-else. This is because, semantically, $G_c$ appears in a negated context. It is all right, however, for $G_c$ to instantiate variables that are used by $G_t$.

**Note!** Execution can backtrack into $G_c$. For example,

```prolog
( if small_prime(X), X > 2 then Y = X * X else Y = -1 )
```

has solutions $Y = 9$, $Y = 25$, and $Y = 49$.

**Prolog programmers take note:** unlike Mercury, Prolog programs commit to the first solution of $G_c$. The Prolog equivalent of the above goal would have $Y = 9$ as its only solution, not $Y = 25$ or $Y = 49$.

**Note!** $(G_c \rightarrow G_t ; G_e)$ is an alternative, albeit old-fashioned, syntax for $(if G_c then G_t else G_e)$.

### 3.4 Procedures and code reordering

The aim of this section is to give the reader some understanding of code reordering. This knowledge is not required to write Mercury programs, but it can help the programmer understand mode-related error messages from the compiler.

We will illustrate using the `append` predicate defined in the `list` standard library module. The declarative semantics of `append(Xs, Ys, Zs)` is that the list $Zs$ is the concatenation of lists $Xs$ and $Ys$. So `append([1], [2, 3], [1, 2, 3])` is true, but `append([2, 3], [1], [1, 2, 3])` is not.

```prolog
:- pred append(list(T), list(T), list(T)).
:- mode append(in, in, out) is det.
:- mode append(out, out, in) is multi.
append(Xs, Ys, Zs) :-
```
CHAPTER 3. THE MERCURY MODE SYSTEM

(  
Xs = [], Zs = Ys  
;  
Xs = [X | Xs0], append(Xs0, Ys, Zs0), Zs = [X | Zs0]  
).

This code needs no reordering for the \((\text{in}, \text{in}, \text{out})\) is \textit{det} mode, where \(Xs\) and \(Ys\) start off instantiated and \(Zs\) starts off uninstantiated. The first disjunct works like this:

1. \(Xs = []\) Deconstruct \(Xs\)  
2. \(Zs = Ys\) Assign \(Zs\)

and the second disjunct works like this:

1. \(Xs = [X | Xs0]\) Deconstruct \(Xs\), instantiating \(X\) and \(Xs0\)  
2. \(\text{append}(Xs0, Ys, Zs0)\) Call the \((\text{in}, \text{in}, \text{out})\) procedure, instantiating \(Zs0\)  
3. \(Zs = [X | Zs0]\) Construct \(Zs\)

Because each disjunct deconstructs \(Xs\) in a different way, this disjunction is a switch. Because the switch is exhaustive, and the other goals in each disjunct are \textit{det}, the switch as a whole is \textit{det}.

The \((\text{out}, \text{out}, \text{in})\) is \textit{multi} mode, where only \(Zs\) is initially instantiated, does require some reordering in order to ensure that every variable is instantiated before it is needed. The first disjunct becomes

1. \(Xs = []\) Construct \(Xs\)  
2. \(Zs = Ys\) Assign \(Ys\)

and the second disjunct becomes

1. \(Zs = [X | Zs0]\) Deconstruct \(Zs\), instantiating \(X\) and \(Zs0\)  
2. \(\text{append}(Xs0, Ys, Zs0)\) Call the \((\text{out}, \text{out}, \text{in})\) procedure, instantiating \(Xs0\) and \(Ys\)  
3. \(Xs = [X | Xs0]\) Construct \(Xs\)

Since this disjunction is not a switch and the first disjunct always leads to a solution, the disjunction as a whole is \textit{multi} in this case.

\textbf{Note!} The Mercury compiler reorders code as little as possible. However, programmers should not write code that depends upon any particular order of evaluation — code can also be reordered by various optimizations! In particular, it
is a bad idea to write something like \(( \text{if } X \neq 0, Z = Y / X \text{ then } \ldots \text{ else } \ldots )\), assuming that the test for \(X\) being non-zero will guarantee that this code cannot lead to a division-by-zero error at run-time. It is certainly possible that the test and the division may be compiled the other way around.

### 3.5 Insts and subtypes

So far we have only talked about variables going from being uninstantiated to being instantiated. It turns out to be useful to also keep track of the possible values a variable can have when it is instantiated. Mercury uses \textit{insts} for this purpose. An \textit{inst} represents the possible instantiation states of a variable at a particular point in a program.

The most basic \textit{insts} are \textit{free}, meaning a variable is uninstantiated, and \textit{ground}, meaning a variable is instantiated with some unknown value of the appropriate type.

The built-in \textit{modes} \textit{in} and \textit{out} are defined using the following syntax:

\begin{align*}
:- \text{ mode in} & \quad == \ (\text{ground} \ >> \ \text{ground}). \\
:- \text{ mode out} & \quad == \ (\text{free} \quad >> \ \text{ground}).
\end{align*}

That is, an \textit{in} mode argument of a goal must be \textit{ground} (i.e., be instantiated with some value) before the goal is executed and will also be \textit{ground} afterwards, while an \textit{out} mode argument must be \textit{free} (i.e., not instantiated) before the goal is executed, but will be \textit{ground} afterwards.

\textit{Note!} If a goal fails or backtracks then the \textit{insts} of its arguments stay the same as they were before the goal was tried.

#### Specialised insts

It is occasionally useful to define new \textit{insts} matching only subsets of possible values that a variable might have. Consider the \textit{inst non_empty_list} which is defined in the \textit{list} standard library module:

\begin{verbatim}
:- inst non_empty_list == bound([ground | ground]).
\end{verbatim}
:- pred head(list(T), T).
:- mode head(in, out) is semidet.
:- mode head(in(non_empty_list), out) is det.

head(Xs, X) :- Xs = [X | _].

The \texttt{inst} declaration defines \texttt{non_empty_list} to mean “bound to the list data constructor \([|]\) whose first argument has \texttt{inst} \texttt{ground} and whose second argument has \texttt{inst} \texttt{ground}”.

The first mode for \texttt{head} tells us that if all we know about the first argument is that it is \texttt{ground} (i.e., it could be bound to any value of type \texttt{list(T)}, including \([]\)) then a call to \texttt{head} is semidet.

The second mode for \texttt{head} says that if we know the first argument is a non-empty list (i.e., whatever value it has, it’s top-most data constructor must be \([|]\) with two \texttt{ground} arguments) then a call to \texttt{head} is guaranteed to succeed.

The second \texttt{mode} declaration uses the built-in parameterised form of the \texttt{in} argument mode, which is defined like this:

):- mode in(I) == (I >> I).

where \(I\) is an \texttt{inst} parameter. \texttt{in(non_empty_list)} is therefore equivalent to writing \((\texttt{non_empty_list} \gg \texttt{non_empty_list})\). There is also a built-in parameterised \texttt{out} argument mode, defined thus:

:- mode out(I) == (free >> I).

When compiling the procedure for the second mode of \texttt{head}, the Mercury compiler uses the information about the \texttt{inst} of the first argument, \(X_s\), to infer that the goal \(X_s = [X | _\] must (a) be a deconstruction and (b) must succeed because whatever value \(X_s\) has matches the pattern \([_|_]\).

\textbf{Note!} A value with a \texttt{bound(...)} \texttt{inst} can always be used in a context where a \texttt{ground} value is expected, but not the other way around.

\textbf{XXX} Mention the alternative \texttt{inst} definition syntax.
Recursively defined insts

It is possible to describe quite complicated instantiation states. The following insts, for instance, describe lists of even and odd lengths respectively:

```prolog
:- inst even_length_list == bound([], [ground | odd_length_list]).
:- inst odd_length_list == bound([ground | even_length_list]).
```

The first inst declaration defines `even_length_list` to mean “bound either to [] or to [|] with two arguments, the first having inst `ground` and the second having inst `odd_length_list` (multiple possibilities in a `bound` expression are separated by semicolons).

The second inst declaration defines `odd_length_list` to mean “bound to [|] with two arguments, the first having inst `ground` and the second having inst `even_length_list`.”

Partial instantiation

A partially instantiated value is one whose inst is `bound(...)` where the ... part contains free sub-insts, either directly or indirectly.

Partial instantiation is not currently supported for several reasons, including the difficulty of analysing such code, the difficulty of maintaining such code, and the difficulty of compiling such code efficiently.

3.6 Uniqueness

A `bound(...)` inst is said to be shared — that is, it corresponds to a value that may be referred to, directly or indirectly, by more than one variable at a given point in the program.

Mercury has a special inst, unique, which is like ground, but it means that there is precisely one reference to the unique data at this point in the program. The counterpart to unique, is clobbered. A variable with inst clobbered may never be used again (e.g., because the value it refers to is now out-of-date or has been overwritten with something else).
CHAPTER 3. THE MERCURY MODE SYSTEM

The most common use of uniqueness is for managing IO. All the IO operations defined in the io standard library module include two arguments of type io, with modes di and uo respectively. di stands for “destructive input” and uo stands for “unique output”. These modes are built-in and defined thus:

```plaintext
:- mode di == (unique >> clobbered).
:- mode uo == (free   >> unique).
```

To illustrate, consider these pred declarations taken from the io module:

```plaintext
:- pred io.write_string(string::in, io::di, io::uo) is det.
:- pred io.write_int(int::in, io::di, io::uo) is det.
:- pred io.nl(io::di, io::uo) is det.
```

and the following code snippet:

```plaintext
io.write_string("The meaning of life is ", IO0, IO1),
io.write_int(42, IO1, IO2),
io.nl(IO2, IO3)
```

The io type arguments denote “states of the world”. These io states are updated when IO actions are performed. One can never go back to an earlier state (you can’t unplay a piece of music or unprint a document), so each IO action clobbers the io state passed to it and produces and new io state as its result. Similarly, because one cannot copy the state of the world, io states have to be unique. These constraints ensure that the above code snippet executes in the expected order — that is, first the string ‘The meaning of life is ’’ will be printed (clobbering IO0 and producing IO1), then the number 42 (clobbering IO1 and producing IO2), and finally a newline (clobbering IO2 and producing IO3).

Say we were to accidentally reuse IO0 in the second goal:

```plaintext
io.write_string("The meaning of life is ", IO0, IO1),
io.write_int(42, IO0, IO2),
io.nl(IO2, IO3)
```
3.7. HIGHER-ORDER MODES

The Mercury compiler will report the following error (line 27 in file foo.m is the call to `io.write_string`):

```
foo.m:027: In clause for 'main(di, uo)'.
foo.m:027: in argument 2 of call to predicate 'io.write_string/3':
foo.m:027: unique-mode error: the called procedure would clobber
foo.m:027: its argument, but variable 'IO0' is still live.
```

**Note!** Procedures that can clobber arguments must have determinism category det or cc_multi. They must always succeed and produce a single result. The reason for this is that once an argument is clobbered, which could happen at any point during the execution of the procedure, there is no way of unclobbering it on failure or backtracking. Consequently it is also an error for code to backtrack into such procedures. For the rare cases where one needs to do such things, the reader is referred to the section on backtrackable destructive update in the Mercury reference manual which discusses “mostly uniqueness”.

Other modules in the standard library that use uniqueness are array and store. The array module implements arrays with O(1) look-up and set operations. It achieves this by using destructive update for the array set operation. This is quite safe because arrays are unique: the ‘old’ version of the array is clobbered by the update operation so it can never be referred to again; the ‘new’ version of the array is simply the updated ‘old’ version. The store module allows one to construct safe, pointer-based structures. Pointer referents can be accessed and updated in O(1) time by using the same technique as for arrays. Chapter XXX describes these modules in more detail.

### 3.7 Higher-order modes

XXX Fill this out.

### 3.8 Committed-choice nondeterminism

XXX Fill this out.
The \texttt{cc\_nondet} and \texttt{cc\_multi} modes. There may be multiple solutions to a \texttt{cc} predicate, but you will only get one of them.

The compiler will report an error if a program can backtrack into a committed-choice goal: all goals following a committed-choice goal must be \textit{guaranteed} to succeed. Programming under this restriction is quite burdensome.

If all solutions to a committed-choice predicate are equivalent, in the sense that, no matter which solution you get, the observable behaviour of your program will be the same, then you can use the built-in function \texttt{promise\_only\_solution} to escape from the committed-choice context.